**Design Notes**

**Interpolation of Data**

This project shows how interpolation of data works in the AWRDE. Interpolation between data points is needed for graphing and simulation purposes. Interpolation can be of three types: linear, spline, and rational. All three cases are demonstrated.

**Overview**

Interpolation between data points occurs in graphing and simulation. Normally, the designer wishes the graph to have a solid line between data points, which requires the software to guess (interpolate) where the line should be drawn. When a simulation is run, the software might require data from another source, for example an imported S parameter dataset, or S parameters from an EM simulation. If the frequencies of the dataset do not match those of the simulation, interpolation will occur.

The designer can set the interpolation type at the global level, under **Options > Project Options > Interpolation/Passivity**, and at the local level for any S parameter dataset under its **Options > Interpolation/Passivity**. There are three settings:

- **Linear**: A straight line is drawn between data points. Either Cartesian or polar coordinates (radius and angle) can be selected.
- **Spline**: A polynomial of degree N is drawn between every two data points. The higher the degree of polynomial, the smoother is the fitting between adjacent curves.
- **Rational**: A curve between points is drawn by approximating the data as a number of poles in the complex plane.

This example shows the three options used on example data. The S parameter dataset used has seven data points. This is graphed in various ways using interpolation to show the differences in the options.

**Data Files**

There are three folders in Data Files section of the Project Browser: "Linear", "Rational", and "Spline". Each folder contains two or more data files. The data in all the data files are the same; each contains one port S parameter data, consisting of seven frequencies, equally spaced between 1 and 42 GHz. The difference between the files is how interpolation is carried out when the data sets are used in other measurements. The interpolation setting for any of the data files can be seen by right clicking on the data file icon, and going to **Options > Interpolation/Passivity**.

The data files are named to correspond to the setting of the interpolation. For example, "Spline_Cartesian" uses Spline interpolation with Cartesian coordinates. There are three types of interpolation: linear, spline, and rational. Each type can either be carried out on a Cartesian coordinate system, or a polar coordinate system. Spline approximation has a setting for "Spline Order", which is a number in the range of 2 to 8. This number has been set to 2, which is a low order, for all spline interpolation, except for the data set: "Spline_Polar_High_Order", where the order is set to 8. Rational approximation has a setting for Window Size, which must be in the range of 2 to 100. The rational approximation data sets have this window size set to 4.

The data were generated from a one port circuit, which is a series RL load, with R = 20 Ohms, and L = 1nH.

**Schematics**

The schematics are organized in the same way as the datasets. Each schematic has one of the datasets in it as a sub-circuit, with one port attached. The schematics are all simulating at the project frequencies, as set in **Options > Project Options > Frequencies**. The frequencies are 1 to 42 GHz in steps of 0.1 GHz. When the schematics are used as data sources for the measurements, the software will have to interpolate the dataset data to get data points at all the project frequencies.

**Measurements**

All the graphs except for the raw data are contained in one of four measurement folders: Linear Approximations, Rational Approximations, Spline Approximations, and Y and Z parameters.

**1_Raw_Data**
This measurement shows S11 for 4 of the datasets on a Smith Chart. Notice there are 7 data points, and that all the curves look exactly the same. This is because there is no interpolation going on here. Raw data is being plotted, and the points are being shown. Straight lines can be drawn between the points if desires by modifying the graph's properties. Right click anywhere on the graph, and select **Properties > Traces**. Then select the trace, and change the line from "none" to the desired line type. It must be emphasized that the straight lines that appear is not interpolation in the sense we are discussing in this example. The software is just drawing lines between data points for visual purposes. Interpolation occurs when the data sets are being used as sub-circuits in other simulations.

### 2 Linear Cartesian

The graph shows how linear interpolation works. The raw data for the real and imaginary parts of S11 are plotted as seven symbols (triangles and boxes) without any connecting lines. The "other traces use linear interpolation to show the data at 450 frequencies (1 to 46 GHz in steps of 0.1 GHz). The traces have been set to be solid, with no symbols. (This is set in the graph's properties: **Properties > Traces**). The Cartesian traces show the expected behavior of straight lines being drawn between the original 7 data points. The polar traces use linear interpolation, put on the magnitude and angle of the S parameter, not on the real and imaginary parts of the S parameter. They therefore are not straight lines in this graph. This is particularly noticeable in the lower frequencies of the real part of S11.

### 3 Linear Polar

This graph shows the same traces as the previous graph, but on a polar plot. The original 7 data points are shown by the triangles. The interpolated points are in straight line segments between the original data points in the Cartesian setting. The polar setting linearly varies the radius and angle of an arc between the original data points. This normally would be the better choices for this graph. Notice that if the magnitude of S11 is plotted in a rectangular graph using the Cartesian interpolation, the resulting traces will have a number of dips in it. This is because the magnitude of S11 is the distance from the center of the polar graph, and this distance varies unevenly as the trace is drawn.

### 4 Spline Cartesian and 5 Spline Polar

These graphs show how spline interpolation works. Splines are higher order polynomials (quadratic, cubic, quartic, ...) that are used instead of straight line interpolation. The order of the spline can be set in the interpolation options. The splines shown here are of order 2, which is quadratic. This is the lowest possible order for a spline. A quadratic has three unknowns. It is required that for any interpolation section that the two end points are the same as the original two data points, and that the derivative on the left end point match the that of the previous section. In this way, the spline is "smooth"; that is, a derivative exists at all points. Higher order splines have higher order derivatives matching at the end points of sections, leading to smoother curves. The maximum order spline supported is 8. The splines shown here use Cartesian and polar coordinate systems. As expected, the Cartesian splines work better on the real and imaginary parts of S11, and the polar splines work better on in the polar graph, which emphasizes the magnitude and angle of S11.

### 6 Rational Cartesian and 7 Rational Polar

These graphs show how rational interpolation works. Rational interpolation models the S parameter data using poles in the complex S plane. It has the advantages of being everywhere smooth. The order of the rational approximation has been set to 4. This is the minimum recommended value to use, although the software permits an order of 2. Rational interpolation gives about the same results as spline of the same order. Rational interpolation was developed to export S parameter data to time domain simulators, for example HSpice, but that discussion is outside the scope of this article.

### 8 Z Cartesian, 8 Z Polar, 9 Y Cartesian, and 9 Y Polar

For completeness, these graphs are included. They show the Y and Z parameters instead of S parameters. The most noticeable feature is that linear interpolation has a hard time following the Z and Y parameters. This is because, transforming from S to Y or Z is not linear. Therefore, a straight line for an S parameter is not a straight line for a Y or Z parameter. For this reason, polar or rational approximation is preferred when working with Z and Y parameters. Measurements in the graphs may be disabled to improve readability. The measurements can be activated/deactivated by right clicking on the measurement in the browser and selecting **Toggle Enable**.
Graph - 7_Rational_Polar